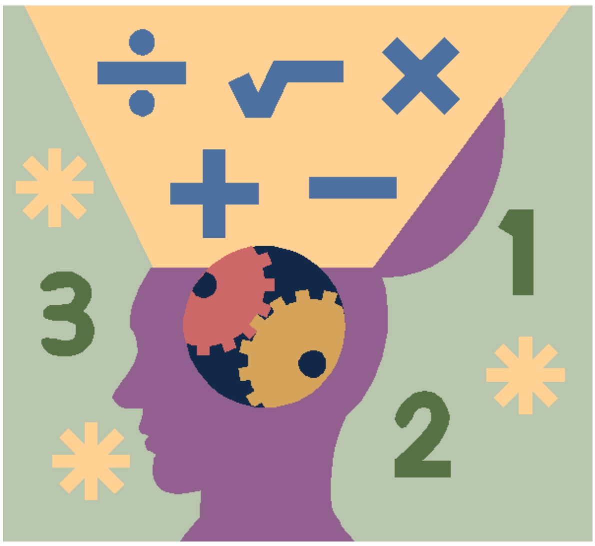
Stirling High School Numeracy Guide



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Introduction

What is "Numeracy"?

According to Learning and Teaching Scotland:-
"Being numerate involves developing a confidence and competence in using number that allows individuals to solve problems, interpret and analyse information, make informed decisions, function responsibly in everyday life and contribute effectively to society."

CfE Numeracy Outcome Areas

The Numeracy Outcomes have been categorised into 8 broad "organisers" under two headings.

NUMBER, MONEY, MEASURE

- Estimation and Rounding
- Number and Number Processes
- Fractions, Decimals and Percentages
- Money
- Time
- Measure

INFORMATION HANDLING

- Data & Analysis
- Ideas of Chance and Uncertainty

Children will develop skills in these organisers as they pass through different "levels" in their education.

It is generally accepted that "Numeracy" is a sub-set of Mathematics but with the advent of the Curriculum for Excellence, all staff are now responsible for the delivery of "Numeracy".

What is the purpose of this booklet?

This booklet has been produced to give guidance to pupils and parents on how certain Numeracy topics are taught in Mathematics and throughout the school. Staff from all departments have been consulted during its production and have been issued with a copy of the booklet. It is hoped that using a consistent approach across all subjects will make it easier for pupils to progress.

How can it be used?

If you are helping your child with their homework, you can refer to the booklet to see what methods are being taught in school. Look up the relevant page for a step by step guide.

Why do some topics include more than one method?

In some cases (eg percentages), the method used will be dependent on the level of difficulty of the question and whether or not a calculator is permitted.

For mental calculations, pupils should be encouraged to develop a wide variety of strategies so that they can select the most appropriate method in any given situation. If there are still questions unanswered the Maths department are more than happy to offer advice or support on any aspect of Numeracy.

It is hoped that use of the information in this booklet may help lessen confusion through a more consistent approach to Numeracy; allowing you to better support your child and ultimately lead to an improvement in their understanding and attainment.

Estimating: Measure

Depending on their level of ability and progress we expect pupils to:-

- estimate height and length in cm, m, $\frac{1}{2}$m, $\frac{1}{10}$m

  (possibly by comparison ie if the length of a ruler is 30cm estimate the length of a pencil)

  e.g. length of a pencil is approximately 10cm

  the width of a desk is approximately $\frac{1}{2}$m

- estimate small weights, small areas, small volumes (including by comparison)

  eg a bag of sugar weighs approximately 1kg

- estimate areas in square metres, lengths in mm and m

  eg the area of a blackboard is approximately $4m^2$

  the diameter of a 1p coin is approximately 15mm

- solve practical problems by applying knowledge and understanding of measure, choose an appropriate unit and degree of accuracy for the task in hand, and appreciate the practical importance of accuracy when making calculations.

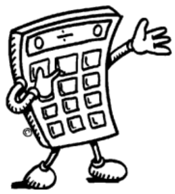
  eg If an angle had to be drawn to $\pm$ 2 degrees. What would be the largest and smallest acceptable angles in drawing 70^0.
  Largest acceptable angle = 72^0 Smallest acceptable angle = 68^0

Estimating: Rounding

Numbers can be rounded to give an approximation.

Depending on their level of ability and progress we expect pupils to:-

- round 2 digit whole numbers to the nearest 10
- round 3 digit whole numbers to the nearest 100
- round any number to the nearest whole number, 10 or 100
- round any number to 1 decimal place
- round to any number of decimal places or significant figures



We always round up from 5 or above

Worked examples:

74 to the nearest 10 → 70

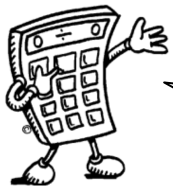
386 to the nearest 10 → 390

347.5 → 348 (to the nearest whole number)
or → 350 (to the nearest ten)
or → 300 (to the nearest hundred)

7.51 → 7.5 (to 1 decimal place)
8.96 → 9.0(to 1 decimal place)

3.14159 → 3.142 (to 3 decimal places)
or → 3.14(to 2 decimal places)
or → 3.14 (to 3 significant figures)

Estimating: Calculations



We can use rounded numbers to give us an approximate answer to a calculation. This allows us to check that an answer is sensible.

Depending on their level of ability and progress we expect pupils to:-

- estimate the cost of a shopping basket by rounding to the nearest pound, to the nearest 50p or pairing to make pounds.
- estimate a calculation by rounding to the nearest whole number, 10 etc.
- estimate a calculation by rounding to one significant figure.

Worked examples:

1. A woman buys various cuts of meat costing £2.39, £5.72 and £1.98 in the butchers. Calculate the total bill.

Estimate = £2 + £6 + £2 = £ 10

Answer = £10.09

Calculate:

$$\begin{array}{r} 2.39 \\ 5.72 \\ +\ 1.98 \\ \hline £10.09 \\ \hline \end{array}$$

2. Tickets for a concert were sold over 4 days. The number of tickets sold each day was recorded. How many tickets were sold in total?

Monday	Tuesday	Wednesday	Thursday
486	205	197	321

Estimate = 500 + 200 + 200 + 300 = 1200

Answer = 1209 tickets

Calculate:

$$\begin{array}{r} 486 \\ 205 \\ 197 \\ +321 \\ \hline 1209 \\ \hline \end{array}$$

3. A bar of chocolate weighs 42g. There are 48 bars of chocolate in a box. What is the total weight of chocolate in the box?

Estimate = 50 x 40 = 2000g

Answer = 2016 g

Calculate:

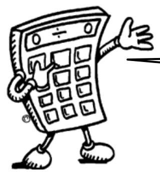
$$\begin{array}{r} 42 \\ \times 48 \\ \hline 336 \\ 1680 \\ \hline 2016 \\ \hline \end{array}$$

Number & Number Processes: Subtraction

As skills progress we;

- Subtract using decomposition (written method).
- Check answers by addition.
- Promote alternative mental methods where appropriate.

Worked examples:
Written Method – Decomposition



WE DO NOT………..
"borrow and pay back!"

eg 271-38

$$\begin{array}{rrr} 2 & {}^{6}\cancel{7} & {}^{1}1 \\ - & 3 & 8 \\ \hline 2 & 3 & 3 \\ \hline \end{array}$$

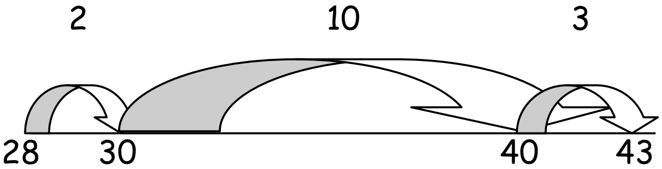
eg 400 - 74

$$\begin{array}{rrr} {}^{3}\cancel{4} & {}^{9}\cancel{0} & {}^{1}0 \\ - & 7 & 4 \\ \hline 3 & 2 & 6 \\ \hline \end{array}$$

Mental Strategies

- Counting On

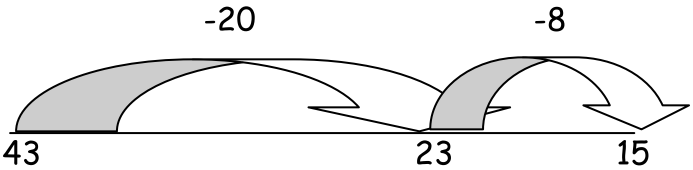
eg To solve 43 – 28, count on from 28 till you reach 43



= 15

- Break up the number being subtracted (partitioning numbers)

eg To solve 43 – 28, subtract 20 then subtract 8



=15

Number & Number Processes: Multiplication & Division 1

Depending on their level of ability and progress we expect pupils to:-

- × and ÷ 2 digit numbers by 2, 3, 4, 5, 10 with the aid of a table square (shown below).
- know their 2x, 3x, 4x, 5x and 10x tables.
- know all of their tables and be able to × and ÷ by 10, 100 1000 (including simple multiples of 10, 100, 1000 eg 20, 300).
- × and ÷ by multiples.
- perform long × and ÷ calculations including those with decimals.

Table Square

x	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10
2	2	4	6	8	10	12	14	16	18	20
3	3	6	9	12	15	18	21	24	27	30
4	4	8	12	16	20	24	28	32	36	40
5	5	10	15	20	25	30	35	40	45	50
6	6	12	18	24	30	36	42	48	54	60
7	7	14	21	28	35	42	49	56	63	70
8	8	16	24	32	40	48	56	64	72	80
9	9	18	27	36	45	54	63	72	81	90
10	10	20	30	40	50	60	70	80	90	100

Mental Strategies

eg Find 39 x 6

Method 1

30 x 6
= 180

9 x 6
= 54

180 + 54
= 234

Method 2

40 x 6
= 240

40 is 1 too many, so take away 6 x 1

240 - 6
= 234

Number & Number Processes: Multiplication & Division 2

Multiplying by multiples of 10 and 100 etc

To multiply by **10** you move every digit **one** place to the left.
To multiply by **100** you move every digit **two** place to the left.

Example 1 (a) Multiply 354 by 10

Th	H	T	U
	3	5	4
3	5	4	0

354 x 10 = 3540

(b) Multiply 50.6 by 100

Th	H	T	U	•	T
		5	0	•	6
5	0	6	0	•	0

50.6 x 100 = 5060

(c) 35 x 30

To multiply by 30, multiply by 3, then by 10

35 x 3 = 105
105 x 10 = 1050

So 35 x 30 = 1050

(d) 436 x 600

To multiply by 600, multiply by 6, then by 100

436 x 6 = 2616
2616 x 100 = 261600

So 436 x 600 = 261600

We may also use these rules for multiplying decimal numbers.

Example 2 (a) 2.36 x 20

2.36 x 2 = 4.72
4.72 x 10 = 47.2

So 2.36 x 20 = 47.2

(b) 38.4 x 50

38.4 x 5 = 192.0
192.0 x 10 = 1920

So 38.4 x 50 = 1920

Number & Number Processes: Multiplication & Division 3

… and to divide by 10 or 100 etc we follow the previous rules but move each digit to the right

Worked Examples

- Example 1

$$\begin{array}{r} 15 \\ \times\ \underline{\ \ 4} \\ \underline{60} \end{array} \qquad \begin{array}{r} 3.4 \\ \times\ \underline{\ \ 3} \\ \underline{10.2} \end{array}$$

- Example 2

$$\begin{array}{r} 234 \\ \times\ \underline{\ \ \ 9} \\ \underline{2106} \end{array}$$

$45 \times 20 = 450 \times 2$
$= 900$

$660 \div 30 = 66 \div 3$
$= 22$

- Example 3

123×24

$$\begin{array}{r} 1\,2\,3 \\ \underline{\times\,2\,4} \\ 4\,9\,2 \\ \underline{2\,4\,6\ \ } \\ \underline{2\,9\,5\,2} \end{array}$$

- Example 4

$4.74 \div 3$

$$\begin{array}{r} 1\,.\,5\ \,8 \\ 3\,\overline{)\,4\,.\,{}^{1}7\,{}^{2}4} \end{array}$$

When dividing a decimal by a whole number, the decimal points must stay in line.

- Example 5

$2.2 \div 8$

$$\begin{array}{r} 0\,.\,2\ 7\,5 \\ 8\,\overline{)\,2\,.\,{}^{2}2\,{}^{6}0\,{}^{4}0} \end{array}$$

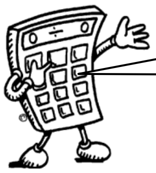
If you have a remainder at the end of a calculation, add a zero to the end of the decimal and continue with the calculation.

Number & Number Processes: Order of Calculations (BODMAS)

Consider this: What is the answer to 2 + 5 x 8?

Is the answer 7 x 8 = 56 or 2 + 40 = 42?

The correct answer is 42.



Calculations which involve more than one operation need to be done in a particular order. The order can be remembered by using the mnemonic **BODMAS.**

The **BODMAS** rule tells us which operations should be done first.

BODMAS stands for:

(B)rackets
(O)rder
(D)ivide
(M)ultiply
(A)dd
(S)ubtract

"Order" means a number raised to a power such as $2^3 = 2 \times 2 \times 2 = 8$

Scientific calculators use this rule but some basic calculators may not, so we must take care in their use. **As ever each step of working should always be shown.**

Worked Examples

1) 12 x (3 + 4) BODMAS tells us to do the bracket then multiply.
= 12 x 7
= 84

2) $3 + 5^2$ BODMAS tells us to work out the power then add.
= 3 + 25
= 28

3) 5.04 + 8.4 ÷ 7 BODMAS tells us to divide then add.
= 5.04 + 1.2
= 6.24

Evaluating Formulae Using BODMAS

Depending on their level of ability and progress we expect pupils:-

- to be able to replace letters by values (substitution) in simple formulae and use **BODMAS** rules to work out an answer.
- to construct and use simple formula.

We do not
- Rearrange the formula before starting (too difficult).
- State the answer only. ALL working must be shown.

Worked Examples

1. Find the area of a rectangle which measures 6cm by 8cm.

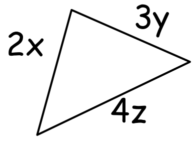
   $A = lb$ when $l = 8$ and $b = 6$
   $A = 8 \times 6$
   $= 48 \text{ cm}^2$

2. Use the formula $I = \frac{V}{R}$ to evaluate I when V = 240 and R = 40.

   $I = \frac{V}{R}$
   $I = \frac{240}{40}$
   $I = 6$

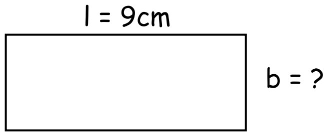
more difficult examples include

3. Find an expression for the perimeter of this shape.



$P = 2x + 3y + 4z$

4. The perimeter of a rectangle is 34cm. If the length is 9cm, what is the breadth?



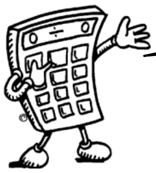
$P = 2l + 2b$ so $34 = 2\times9 + 2b$
$34 = 18 + 2b$
$2b = 16$
$b = 8\text{cm}$

Scientific Notation or Standard Form

In mathematics we introduce Scientific Notation to **some** pupils towards the end of S2. It is also part of the current General and Credit Standard Grade courses and as such is taught at the beginning of S3.

We teach that a number in scientific notation consists of a number between one and ten which is multiplied by 10 to some power.

ie $a \times 10^n$ where $1 \le a < 10$



Worked Examples

1. Write in standard form:

a) 23 500
Answer = 2.35×10^4

b) 124 500 000
Answer = 1.245×10^8

c) 0.000 000 34
Answer = 3.4×10^{-7}

2. Write in normal form:

a) 3.5×10^6
Answer = 3 500 000

b) 4.05×10^{-4}
Answer = 0.000 405

Pupils also learn about powers and square roots and how to use these functions on scientific calculators. For those not achieving this level by the end of S2 this topic is taught at the beginning of S3.

NB
The terms “kilo” meaning one thousand or 10^3 and “milli” meaning one thousandth or 10^{-3} are introduced.

Fractions 1

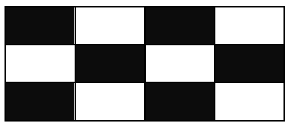
Depending on their level of ability and progress we expect pupils to:-

- understand the meaning of a fraction and be able to find $\frac{1}{2}$ or $\frac{1}{4}$ using concrete materials.
- calculate simple fractions such as $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$ or $\frac{1}{10}$ of 1 or 2 digit numbers.
- calculate fractions such as $\frac{2}{3}$, $\frac{3}{4}$, $\frac{7}{10}$ of numbers up to 4 digits.
- use the equivalences of widely used fractions to find a fraction of a quantity both mentally, using written methods and with a calculator.
- use equivalence of all fractions, decimals and percentages, add subtract, multiply and divide fractions with and without a calculator.

Worked Examples

1. What fraction of the flag is shaded?

Answer = $\frac{6}{12} = \frac{1}{2}$



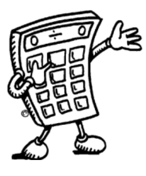
2. $\frac{1}{3}$ of 12 = 12 ÷ 3 = 4

3. $\frac{3}{4}$ of 200

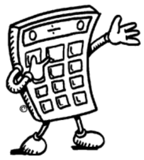
To find $\frac{3}{4}$, start by finding $\frac{1}{4}$ then multiply your answer by 3

$\frac{1}{4}$ of 200 = 50

so $\frac{3}{4}$ of 200 = 3x50 = 150



Simplifying Fractions/Equivalent Fractions



The top of a fraction is called the **NUMERATOR**, the bottom is called **DENOMINATOR.**
To simplify a fraction we simply divide both the numerator and the denominator of the fraction by the same quantity. The answer is a fraction which is **equivalent** to the one you started with.

Example 1 Simplify $\frac{20}{25}$

$$\frac{20}{25} \overset{\div 5}{\underset{\div 5}{=}} \frac{4}{5}$$

This can be done repeatedly until the numerator and denominator are the smallest possible numbers – the fraction is then said to be in its **"simplest form"**.

Example 2 Simplify

$$\frac{72}{84} \overset{\div 2}{\underset{\div 2}{=}} \frac{36}{42} \overset{\div 2}{\underset{\div 2}{=}} \frac{18}{21} \overset{\div 3}{\underset{\div 3}{=}} \frac{6}{7}$$

Example 3 Find the numerator of the second fraction.

$$\frac{3}{5} = \frac{?}{30}$$

×6

This time we have multiplied 5 by 6 to get 30. To find the missing numerator we must also multiply 3 by 6

so $\frac{3}{5} = \frac{18}{30}$

Decimal Fractions

Pupils must be aware of decimal equivalences and be able to work with them in calculations. Some common decimal equivalences are summarised in the section on **"Percentages"**.

Example 1 Calculate 0.3 of £120.

$\frac{3}{10}$ of £120 = (120 ÷10) x 3

= 12 x 3

= £36

0.3 = $\frac{3}{10}$, we then use this in our calculation

Fractions 2

- Once fundamental skills have been mastered some pupils progress to Add/Subtract, Multiply and Divide fractions.

Worked Examples

Add/Subtract	**Multiply**	**Divide**
make the denominators equal then + or – only the top numbers	multiply top and multiply bottom	Invert the second fraction then multiply.
$\frac{1}{2}+\frac{1}{3}$ $= \frac{3}{6}+\frac{2}{6}$ $= \frac{5}{6}$	$\frac{2}{3}\times\frac{3}{4}$ $= \frac{6}{12}$ $= \frac{1}{2}$	$\frac{3}{4}\div\frac{2}{5}$ $= \frac{3}{4}\times\frac{5}{2}$ $= \frac{15}{8}$ $= 1\frac{7}{8}$

If the question contains a mixture of whole numbers and fractions, the whole numbers should be converted to a "top heavy" fraction before proceeding with the calculation.

Example $4\frac{2}{3}\times 2\frac{1}{2}$

$= \frac{14}{3}\times\frac{5}{2}$

$= \frac{70}{6}$

$= 11\frac{4}{6}$

$= 11\,\frac{2}{3}$

$4\frac{2}{3}$ can be written as $\frac{14}{3}$, a top heavy fraction and similarly $2\frac{1}{2}$ as $\frac{5}{2}$

In all cases top heavy answers should be expressed as mixed numbers and all answers should be written in their simplest form.

Percentages 1

Not all pupils progress to work with percentages. However most will be aware that "percent" means "out of a hundred" and may be familiar with commonly known percentages such as 50%, 10%.

For those progressing to work more fully with percentages, depending on their level of ability and progress we expect pupils to:-

- find 50%, 25%, 10%, 5% etc. without the use of a calculator by using equivalence fractions, multiples of ten and addition techniques
- find percentages with a calculator by first changing the percentage to a decimal.

and at a more advanced level

- find more complicated fractions mentally ie 12.5% 17.5% etc.
- express a fraction as a percentage via a decimal equivalence.

Common Percentages
Some pupils find it useful to learn the common percentages listed in the table below.

Percentage	Fraction	Decimal
1%	$\frac{1}{100}$	0.01
10%	$\frac{1}{10}$	0.1
$12\frac{1}{2}$%	$\frac{1}{8}$	0.125
20%	$\frac{1}{5}$	0.2
25%	$\frac{1}{4}$	0.4
$33\frac{1}{3}$%	$\frac{1}{3}$	0.333…..
50%	$\frac{1}{2}$	0.5
$66\frac{2}{3}$%	$\frac{2}{3}$	0.666….
75%	$\frac{3}{4}$	0.75

Percentages 2 (Mental Strategies)

There are many ways to calculate percentages of a quantity. The methods we use are shown below.

Worked Examples

Method 1 Using Equivalent Fractions

1. Express $\frac{2}{5}$ as a percentage.

$\frac{2}{5} = \frac{4}{10} = \frac{40}{100} = 40\%$

2. Find 25% of £640

25% of £640 = $\frac{1}{4}$ of £640 = £640 ÷ 4 = £160

Method 2 Using 1%

In this method, first find 1% of a quantity (by dividing by 100), then multiply to give the required value.

3. Find 9% of 200g

1% of 200g = $\frac{1}{100}$ of 200 = 200 ÷ 100 = 2g

so 9% of 200g = 9 x 2g = 18g

Method 3 Using 10%

This method is similar to the one above. First find 10% of a quantity (by dividing by 10), then multiply to give the required value.

4. Find 70% of £35

10% of £35 = $\frac{1}{10}$ of £35 = £35 ÷ 10 = £3.50

so 70% of £35 = 7 x £3.50 = £24.50

Percentages 3 (Mental Strategies cont.)

The previous 2 methods can be combined so as to calculate any percentage.

Worked Examples

5. Find 23% of £15000

   10% of £15000 = £1500   so 20% = £1500 x 2 = £3000
   1% of £15000 = £150   so 3% = £150 x 3 = £450

   23% of £15000 = £3000 + £450 = £3450

Finding VAT (without a calculator)

Value Added Tax (VAT) can vary
Previously it was 15%, 17.5% and is currently 20%.

We have already seen how to find 20%.

VAT at 15% - to calculate find 10%, then half this answer to find 5%

VAT at 17.5% - to calculate find 10% then half this answer to find 5% then half this answer again to find 2.5%

6. Calculate the total price of a computer which costs £650 excluding VAT.

   10% of £650 = £65 (to find 10% divide by 10)
   5% of £650 = £32.50 (half the previous answer £65)
   2.5% of £650 = £16.25 (half the previous answer £32.50)

   17.5% of £650 = £65 + £32.50 + £16.25 = £113.75

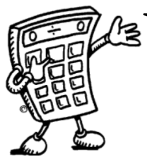
   so the total price = £650 + £113.75 = £763.75

Percentages 4 Using a Calculator

To find the percentage of a quantity using a calculator, change the percentage to a decimal, then multiply.

Worked Example

1. Find 23% of £15000
   23% = 0.23
   so 23% of £15000 = 0.23 x £15000 = £3450



We do not use the % button on calculators because of inconsistencies between models.

Expressing Quantities as a Percentage

To find a percentage, first make a fraction, then convert it to a decimal by dividing the top number (numerator) by the bottom number (denominator) in the fraction. This answer can then be multiplied by 100 to convert the decimal to a percentage.

Worked Examples

1. There are 30 pupils in class 3D - 18 are girls.
   What percentage of the class are girls?

   $\frac{18}{30}$ = 18 ÷ 30 = 0.6 0.6 x 100 = 60% so $\frac{18}{30}$ = 60%

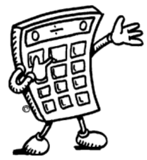
2. James scored 36 out of 44 in a class biology test.
   What is his percentage mark?

   $\frac{36}{44}$ = 36 ÷ 44 = 0.81818…. 0.81818….x100 = 81.818….%

   so $\frac{36}{44}$ = 82% (rounded to the nearest whole number)

Ratio & Proportion 1

Before going on to look at proportion it is important that pupils have some understanding of ratio.



When quantities are to be mixed together, the ratio, or proportion of each quantity is often given. The ratio can be used to calculate the amounts of each quantity, or to share a total into parts.

Writing Ratios

Worked Example

To make a fruit drink, 4 parts water is mixed with 1 part fruit juice.
The ratio of water to juice is 4:1
(said "4 to 1")
The ratio of fruit juice to water is 1:4

Order is important when writing ratios

Simplifying Ratios

Ratios can be simplified in much the same way as fractions

Worked Example

In a bag of balloons there are 15 pink, 10 blue and 5 yellow balloons.

The ratio of blue to pink to yellow is 10:15:5

This should be simplified by dividing each number in the ratio by a common factor. As 5 is the common factor, the simplified answer becomes 2:3:1.

Using Ratios

The ratio of fruit to nuts in a chocolate bar is 3:2. If a bar contains 15g of fruit, what weight of nuts will it contain?

	Fruit	Nuts	
	3	2	
X 5			X 5
	15	10	

So the bar will contain 10g of nuts

Ratio & Proportion 2

Sharing in a Given Ratio

Worked Example

Lauren and Sean earn money by washing cars. By the end of the day they have made £90. As Lauren did more of the work, they decided to share profits in the ratio 3:2. How much money did each receive?

Step 1 Add up the numbers to find the total number of parts

$3 + 2 = 5$

Step 2 Divide the amount you want to share by this number to find the value of each part.

$90 \div 5 = £18$ so each part = £18

Step 3 As the profit is shared in the ratio 3:2, Lauren gets 3 parts and Sean gets 2 parts.

Lauren gets £18 x 3 = £54
Sean gets £18 x 2 = £36

Step 4 Check that the total is correct

£54 + £36 = £90 ✓

So Lauren received £54 and Sean received £36

Proportion

Depending on their level of ability and progress we expect pupils to:-

- use concrete materials and pictures to support learning in this topic.
- calculate the cost of 1 item given the cost of a number of similar items.
- work with simple direct proportion.
- identify direct and inverse proportion, use the unitary method (i.e. find the value of "one" first then multiply by the required value) to find required value.

It is good practice to set working out in a table.

Worked Examples

1. If 6 identical books cost £12.60. What is the cost of 1 book?

	No of Books	Cost	
÷ 6	6	£ 12.60	÷ 6
	1	£ 2.10	

1 book costs £2.10

2. If 5 footballs cost £14.75, how much will 9 footballs cost?

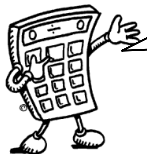
		No of Footballs	Cost	
find the cost of 1	÷ 5	5	£14.75	÷ 5
then the cost of 9	× 9	1	£2.95	× 9
		9	£26.55	

9 footballs cost £26.55

Proportion continued

Worked Examples continued

Direct Unitary Method (i.e. find the value of "one" first then multiply by the required value) to find required value.



Two quantities are said to be in **direct proportion** if when one doubles the other doubles. Similarly if one halves the other also halves.

3. If 5 bananas cost 80p, what is the cost of 7 bananas?

As the number of bananas increases so does the cost → direct proportion

Using the **Direct Unitary Method**

	No of Bananas	Cost
	5 →	80p
find the cost of 1	1 →	80 ÷ 5 = 16p
then the cost of 7	7 →	16 x 7 = £ 1.12

Two quantities are said to be in **inverse or indirect proportion** if when one doubles the other halves.

4. If 5 men take 12 hours to paint a fence, how long will it take 6 men?

As the number of men increases the length of time to do the job will decrease → inverse or indirect proportion

Using the **Inverse Direct Unitary Method**

No of Men	Time in hours
5	12
1	12 x 5 = 60
6	60 ÷ 6 = 10 hours

NB 1. If rounding is required we do not round till the last stage
2. Always use common sense to determine whether an answer looks ✓

Money

As can be seen from previous topics being skilled with money is embedded throughout the course. However, depending on their level of ability and progress we expect pupils to:-

- use coins up to the value of £1
- work out simple totals, work out change, state coins/notes required to pay for an item – some may require tangible materials such as plastic coins.
- add, subtract, multiply and divide (by a single digit) and calculate simple shopping bills
- compare prices, calculate more complicated shopping bills, work backwards to find a given cost (given the total bill and the cost of the other items), convert sterling to a given currency and convert back to sterling giving answers to the nearest penny.
- understand the meaning of and be able to calculate Profit, Loss, Hire Purchase, Salaries (including overtime, time and a half, double time, monthly, annual etc) VAT.
- at a more advanced level demonstrate an understanding of budgeting, personal finance, household finance (Including Compound Interest, Appreciation/ Depreciation, % Profit/Loss, Finding original costs given a sale price, Insurances etc)
- At **Standard Grade G/F** understand the meaning of and be able to calculate Profit, Loss, Hire Purchase, Salaries (including overtime, time and a half, double time, monthly, annual etc) VAT, Car costs, Personal finance such as credit/debt cards etc

When using a calculator we **ALWAYS** show each step of working and if rounding is required, we **DO NOT ROUND** calculations till the last step.

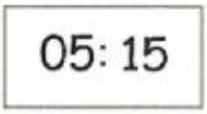
Time Calculations

Depending on their level of ability and progress we expect pupils to:-

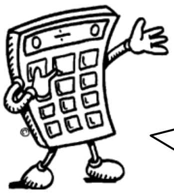
- read digital clocks and be able to identify half past and quarter past the hour on analogue clocks.
- appreciate 12 hour time and calculate time intervals which are less than an hour.
- convert between 12 hour times and 24 hour clock
  2208 = 10.08 pm
- calculate the duration in hours and minutes by drawing a time line or by counting up to the next hour then on to the required time, convert between hours and minutes (multiply by 60 to change hours to minutes)
- use decimals for time in seconds ie 3 minutes 23.4 seconds
- express hours and minutes as a decimal fraction of an hour

Worked Examples:

1.



These clocks both show fifteen minutes past five or quarter past 5.



When writing times in 12 hour clock we must add am or pm after the time.
am is used for times between midnight and 12 noon (morning)
pm is used for times between 12 noon and midnight (afternoon)

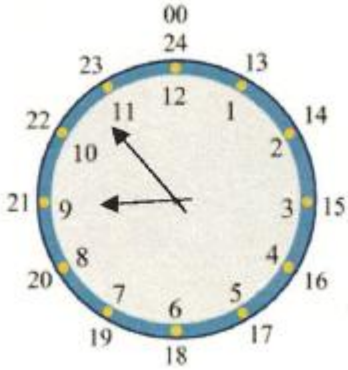
2. How long does a TV show last if it starts at 4.15 pm and finishes at 4.45 pm.

Pupils will count on from 4.15 to 4.45 to find 30 minutes.

Time Calculations

24-Hour Clock

In 24 hour clock, the hours are written as numbers between 00 and 24. Midnight is expressed as 0000 or 2400. After 12 noon, the hours are numbered 13, 14, 15, etc.



Examples

9.55 am ⟶ 09 55 hours
3.35 pm ⟶ 15 35 hours
12.20 am ⟶ 00 20 hours
02 16 hours ⟶ 2.16 am
20 45 hours ⟶ 8.45 pm

Worked Examples

3. How long is it from 0820 to 1625

	0820 to 0900	=		40 min
	0900 to 1600	=	7 hrs	
	1600 to 1625	=		25 min
so	0820 to 1625	=	7 hrs	65 min

but 65 min is 1 hour 5 minutes

therefore 0820 to 1625 = 8 hours 5 minutes

WE DO NOT teach time as a subtraction

4. What time is three tenths of a second more than 24.5 seconds?

$\frac{3}{10}$ = 0.3 so 24.5 + 0.3 = 24.8 seconds

5. Change 24 minutes into a decimal fraction of an hour.

24 min = $\frac{24}{60}$ hours = 24 ÷ 60 hours = 0.4 hours

Information Handling – Frequency Tables

It is sometimes useful to display information in graphs charts or tables. The most common are outlined below.

Frequency Tables – Data is represented by tally marks. Tally marks are grouped in 5's to make them easier to read and count. They are then stated as a frequency. A lot of data is sometimes grouped in intervals.

Worked Examples

1. A class had the following shoe sizes 5 5 5 6 4 3 5 6 6 6 7 4 7 5 6 7 6. Express this information in a frequency table.

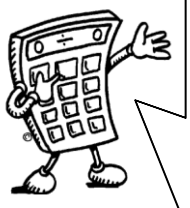
Shoe size	Tally	Frequency
3	I	1
4	II	2
5	~~IIII~~	5
6	~~IIII~~ I	6
7	III	3

2. Draw a frequency table showing the Homework marks for 4B

27 30 23 24 22 35 24 33 38 43 18 29 28 28 27
33 36 30 43 50 30 25 26 37 35 20 22 24 31 48

Mark	Tally	Frequency
16 - 20	II	2
21 - 25	~~IIII~~ II	7
26 - 30	~~IIII~~ IIII	9
31 - 35	~~IIII~~	5
36 - 40	III	3
41 - 45	II	2
46 - 50	II	2

Information Handling – Bar Graphs



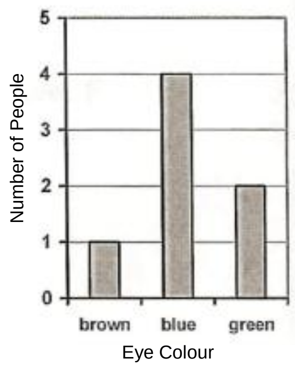
When drawing graphs we expect all pupils to;
- use a sharp pencil and a ruler
- give the graph a title
- label the axes
- label the bars in the centre of the bar (each bar should have an equal width)
- if the data is discrete rather than grouped intervals we leave spaces of equal width between the bars
- label the frequency on the vertical axis

Otherwise, depending on their level of ability and progress we expect pupils to:-

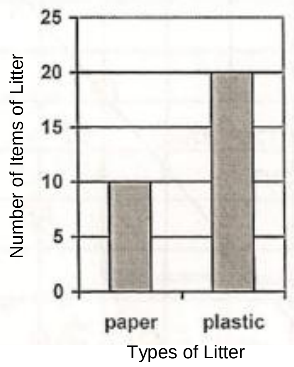
- construct bar graphs with the frequency graduated in single units.
- construct bar graphs with the frequency graduated in multiple units.
- construct bar graphs involving simple fractions or decimals.
- construct a bar graph based on data with grouped intervals or with multiple data.

Worked Examples

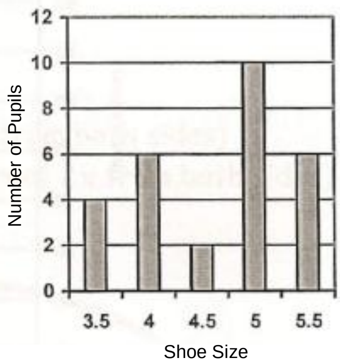
Colour of Eyes

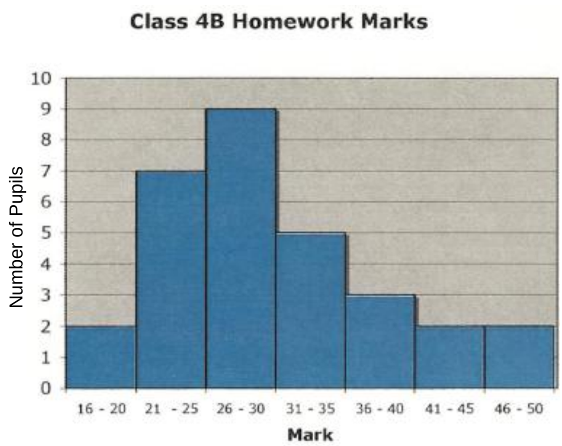


Quantities of Litter

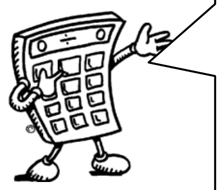


Shoe Size





Information Handling – Line Graphs



Line graphs consist of a series of points which are plotted with a cross or a dot, then joined by a line. Like bar graphs, each line graph should have a title, and each axis must be labelled.
NUMBER THE LINES NOT THE SPACES.
The **trend** is a general description of the graph.

Depending on their level of ability and progress we expect pupils to:-

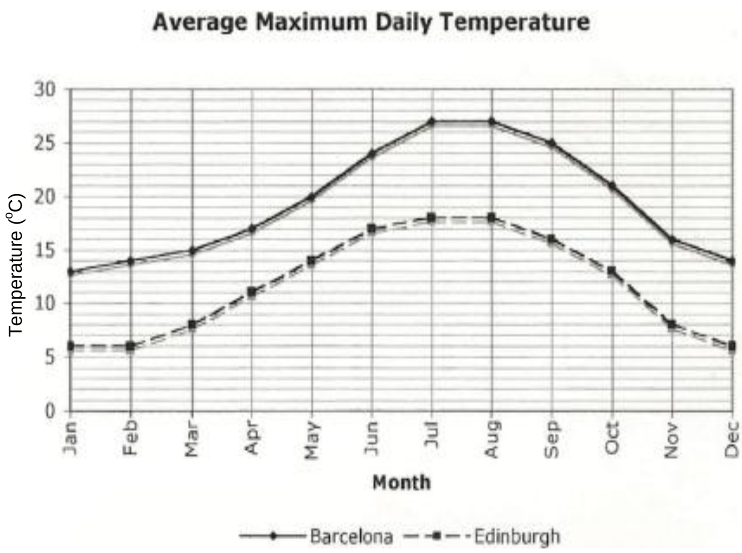
- if necessary, make use of a jagged line to show that the lower part of the graph can be missed out.

Worked Examples

1. Show Heather's weight over 14 weeks as she follows an exercise programme and comment on the general trend of the graph.

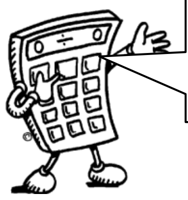
The trend of the graph is that her weight is decreasing.

2. Comment on the trend of the temperatures in Edinburgh and Barcelona.



The trend of the graph is that in both cities the temperatures steadily rise to a maximum in July – August, after this the temperatures fall. The temperature in Barcelona is always warmer than Edinburgh.

Information Handling – Scatter Graphs



A scatter diagram is used to display the relationship between two variables. A pattern may appear on the graph. This is called a correlation.

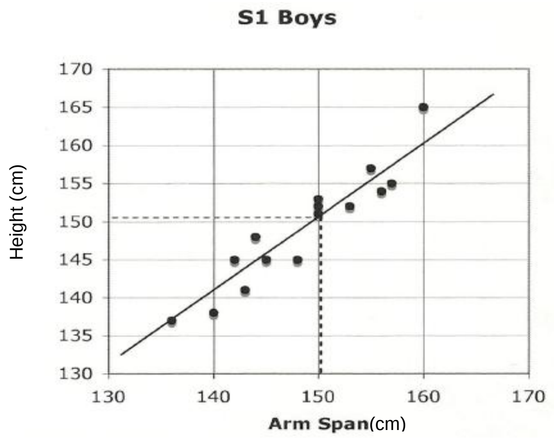
Depending on their level of ability and progress we expect pupils to:-

- plot data on a scatter graph and describe the correlation in qualitative terms such as –
  " the warmer the weather, the more ice-cream is sold" – positive correlation
  or " the warmer the weather, the less you spend on heating" – negative correlation

Worked Example

The table below shows the height and arm span of a group of first year boys. This is then plotted as a series of points on the graph.

Arm Span (cm)	150	157	155	142	153	143	140	145	144	150	148	160	150	156	136
Height (cm)	153	155	157	145	152	141	138	145	148	151	145	165	152	154	137

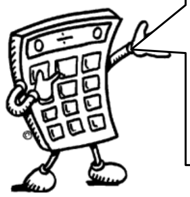


The graph shows a general trend, that as the arm span increases, so does the height. This graph shows positive correlation.

The line drawn is called "the line of best fit". This line can be used to provide estimates. For example a boy with an arm span of 150cm would be expected to have a height of around 151cm.

In some subjects it is a requirement that the axes start from zero.

Information Handling – Pie Charts

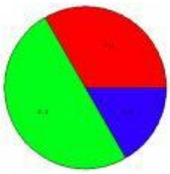


When drawing pie charts we expect all pupils to;
- use a sharp pencil and a ruler
- label all the slices or insert a key as required
- give the pie chart a title

Otherwise depending on level of ability and progress we expect pupils to:-
- interpret simple pie charts.
- construct pie charts involving simple fractions or decimals.
- construct a pie chart of data expressed in percentages using a pie chart scale.
- construct a pie chart from raw data.

Worked Examples

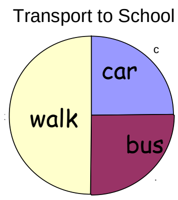
1. The pie chart shows class 2N's "Favourite Colour"



a) What was the least favourite colour? (blue)

b) What fraction of the class liked green? $\left(\frac{1}{2}\right)$

2. $\frac{1}{4}$ of the pupils in a first year class came to school by car, $\frac{1}{4}$ came by bus and $\frac{1}{2}$ walked. Show this in a pie chart.



3. 30% of pupils travel to school by bus, 10% by car, 55% walk and 5% cycle.
Draw a pie chart of the data.
10% of 360° =36°

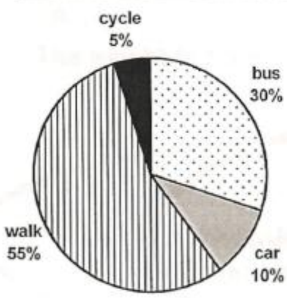
Bus	30%	= 3 x 10% = 108°
Car	10%	= 1 x 10% = 36°
Walk	55%	= 5.5 x 10% =198°
Cycle	5%	= 0.5 x 36% = 18°

4. 20 pupils were asked "What is your favourite subject?"
Replies were Maths 5, English 6, Science 7, Art 2
Draw a pie chart of the data.
360 ÷ 20 (the total) = 18°

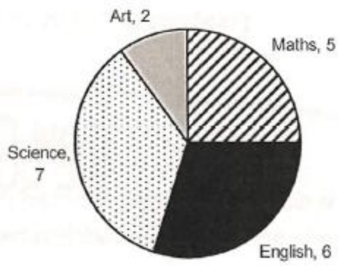
Maths 5	5 x 18	=	90°
English 6	6 x 18	=	108°
Science 7	7 x 18	=	126°
Art 2	2 x 18	=	36°

N.B.
Check the total no. of degrees = 360!

Transport to school



Favourite subject



Information Handling – Data Analysis

Depending on their level of ability and progress we expect pupils to:-

- identify the most/least and make simple comparisons.
- interpret information from a simple display.
- analyse ungrouped data using a tally table and frequency column or ordered list (see frequency table).
- calculate the range of a data set. (difference between the highest and lowest values of data.)
- calculate the mean (average) of a set of data.
- interpret what data conveys and discuss whether the information is robust, vague or misleading.
- use a stem and leaf diagram.
- calculate the mean (average), median (central value of an ordered list), mode (most common value) of a data set.
- find the mode – most frequently occurring value of data
- obtain these values from an ungrouped frequency table
- appreciate the concept of chance and uncertainty
- express probability – always as a fraction in its simplest form

  $$P(\text{event}) = \frac{\text{number of favourable events}}{\text{total number of possible outcomes}}$$

  eg P(even no when a die is rolled) = $\frac{3}{6} = \frac{1}{2}$ (6 no's on die, 3 even)
- use probability to determine expectation in order to make predictions, informed choices and decisions.

Worked Example

The results of a survey of pets pupils owned were:
3 , 3 , 4 , 4 , 4 , 5 , 6 , 6 , 7 , 8.

$$\text{Mean} = \frac{3 + 3 + 4 + 4 + 4 + 5 + 6 + 6 + 7 + 8}{10} = \frac{50}{10} = 5$$

As the data is ordered we can read off the median as the middle value

3 , 3 , 4 , 4 , 4 ↑ 5 , 6 , 6 , 7 , 8

Median

but we have 10 pieces of data so the median will be between 4 and 5
ie. Median = (4 + 5) ÷ 2 = 4.5

Mode = most common = 4

Range = highest – lowest = 8 – 3 = 5

Equations

Although not strictly a Numeracy outcome some may find a note on equations helpful.

Depending on their level of ability and progress we expect pupils to:-

- form simple equations from pictures.
- solve simple equations by "the cover up method"
- solve equations by "balancing" / performing the same operation on each side of the equation.
- solve more involved equations including equations with brackets and/or integers.

Worked Examples:

1. $x + 6 = 10$ To solve using "the cover up method" simply cover up the "x" with your finger and ask "What plus 6 equals 10". The answer must be 4.

   So $x = 4$

2. $2x + 5 = 13$ To balance the equation take 5 away from both sides then divide each side by 2.

   $2x = 8$

   $x = 4$

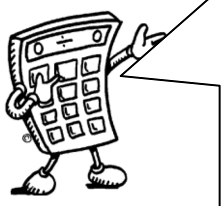
3. $2(3x-2) = x+16$ Multiply out the bracket before balancing

   $6x-4 = x+16$ To balance the equation take away x from both sides,

   $5x-4 = 16$ then add 4 from both sides,

   $5x = 20$ then divide both sides by 5.

   $x = 4$



WE DO NOT CHANGE THE SIDE, CHANGE THE SIGN

We expect;-

- each line of working to be shown
- one equals sign per line
- equal signs below each other
- "*x*"to be written differently from a multiplication sign

We discourage bad form such as 3 x 4 = 12 ÷ 2 = 6 x 3 = 18 and simply guessing an answer

Co-ordinates

Again, not strictly a Numeracy outcome some may find a note on co-ordinates helpful.

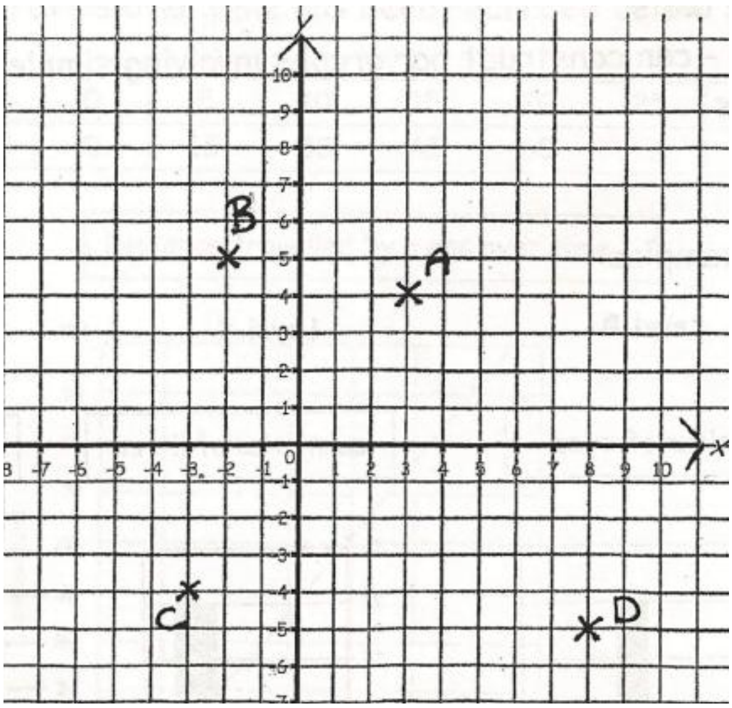
All pupils are expected to have an awareness of grid reference systems in every day contexts and can use them to locate and describe position.

Depending on their level of ability and progress we expect pupils to:-

- use a co-ordinate system to locate a point on a grid
- number grid lines rather than spaces
- use the terms across/back and up/down for the different directions on the x and y axes
- use a comma and brackets to separate and list the coordinate as follows; 3 across, 4 up → (3, 4)
- refer to (0,0) as the origin
- appreciate the historical significance of co-ordinates
- use co-ordinates in all four quadrants to plot/identify position

Worked Example:

Plot the points A(3,4), B(-2,5), C(-3,-4), D(8,-5)



Appendix 1 Mathematical Dictionary (Key Words)

Add; Addition (+)	To combine 2 or more numbers to get one number (called the sum or the total) eg 12 + 76 = 88
am	(ante meridiem) Any time in the morning (between midnight and 12 noon).
Approximate	An estimated answer, often obtained by rounding to nearest 10, 100 or decimal place.
Calculate	Find the answer to a problem. It doesn't mean that you must use a calculator!
Data	A collection of information (may include facts, numbers or measurements).
Denominator	The bottom number in a fraction (the number of parts into which something is split).
Difference (-)	The amount between two numbers (subtraction). eg The difference between 50 and 36 is 14 50 – 36 = 14
Division (÷)	Sharing a number into equal parts eg 24 ÷ 6 = 2
Double	Multiply by 2
Equals (=)	Makes or is the same amount as.
Equivalent fractions	Fractions which have the same value. eg $\frac{6}{12}$ and $\frac{1}{2}$ are equivalent fractions
Estimate	To make an approximate or rough answer.
Evaluate	To work out the answer.
Even number	A number that can be divided by 2 with no remainder. Even numbers end with 0, 2, 4, 6, 8.
Factor	A number which divides exactly into another number, leaving no remainder.
Frequency	How often something happens. In a set of data, this is the number of times a number or category occurs.
Greater than (>)	Is bigger or more than. eg 10 is bigger than 6 $10 > 6$
Least	The lowest number in a group (minimum).
Less than (<)	Is smaller or lower than. eg 15 is less than 21 $15 < 21$
Maximum	The largest or highest number in a group.
Mean	The arithmetic average of a set of numbers (see page 33).
Median	Another type of average – the middle number of an ordered set of data (see page 33).

Minimum	The smallest or lowest number in a group.
Minus (-)	To subtract (take away).
Mode	Another type of average – the most frequent number or category (see page 33).
Most	The largest or highest number in a group (maximum)
Multiple	A number which can be divided by a particular number, leaving no remainder. eg Some of the multiples of 4 are 8, 16, 48, 72.
Multiply (x)	To combine an amount a particular number of times eg 6 x 4 = 24.
Negative number	A number less than zero. Shown by a minus sign.
Numerator	The top number in a fraction.
Odd number	A number which is not divisible by 2 without a remainder. Odd numbers end in 1, 3, 5, 7, 9
Operations	The four basic operations are addition, subtraction, multiplication and division.
Order of operations	The order in which operations should be done. BODMAS (see pages 11 and 12)
Place Value	The value of a digit depending on its place in the number eg in the number 1573.4, the 5 has a place value in the 100 column so it represents 500.
Power	The number of times a number is multiplied by itself. eg 2^3 (2 to the power of 3) = 2 x 2 x 2 = 8
pm	(post meridiem) Any time in the afternoon or evening (between 12 noon and midnight).
Prime number	A number that has exactly 2 factors (can only be divided by itself and 1). Note that 1 is not a prime number as it has only 1 factor and that 2 is the first prime number.
Product	The answer when two numbers are multiplied together eg the product of 5 and 4 is 20.
Remainder	The amount left over when dividing a number.
Share	To divide into equal groups.
Sum	The total of a group of numbers (found by adding)
Square Root	The opposite of "power". Simple roots can be found by asking "what x what gives". More complicated roots have to be calculated on a calculator eg$\sqrt{25}$ = 5 (ie 5x5 = 25).
Total	The sum of a group of numbers (found by adding).